Using the Hungarian Algorithm for Channel Assignment in Wireless Communications

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Abstract

In order to alleviate crowding of the radio frequency spectrum, we have developed a method of spectrum allocation to improve data transmission efficiency. Our method uses the Hungarian Algorithm to maximize the rate of transmission among combinations of user-channel assignments. We present our approach, relevant background information, an experimental and theoretical comparison to fixed allocation, and a discussion of future work.

Executive Summary

Wireless communication devices such as Wi-Fi, cellphones, radios, and numerous IoT enabled appliances/accessories all require access to a portion of the radio frequency spectrum in order to transmit and receive signals. The radio frequency spectrum consists of electromagnetic waves with frequencies ranging from 30 Hz to 300 GHz and is divided into allocated frequency bands, or channels [1]. The use of wireless devices is exploding in terms of the number of devices in use, how heavily they are used, and the amount of data needed to be transferred. Thus, the total amount of data needing to be transmitted is increasing, and the radio frequency spectrum is becoming more and more crowded. Since only one device can occupy a channel at a specific time and location, spectrum crowding poses a growing issue [2].

This project aims to alleviate spectrum crowding by matching devices to channels in a way that maximizes the rate of data transfer, or in other words, the transmission efficiency. With greater transmission efficiency, a given payload will require fewer channels. As a result, the same frequency spectrum can accommodate more devices.

At any given time, many devices need to use the radio frequency spectrum and there are many channels available. Finding the best channel or group of channels for each device becomes a problem of combinatorial optimization—picking the best combination of device-to-channel assignments out of many. This is modeled by a classical problem called the Assignment Problem, where a set of agents must be assigned to a set of tasks. The solution is the combination of agent-to-task assignments where the total cost of the agents performing their tasks is minimized, along with a few other constraints [3].

In our approach, the best combination of device-to-channel assignments is represented by a solution to the Assignment Problem, where agents represent channels, tasks represent devices

to be accommodated, and a low cost represents a high transmission rate. Even though there are many possible combinations of assignments, the Hungarian Algorithm provides a way to obtain an optimal combination without evaluating every single one [4]. For this project, the Hungarian Algorithm was implemented in the Python programming language.

After writing a program to perform our method of device-channel assignment, an experiment was performed to compare it with the simple approach of using fixed allocation. Our approach yielded a transmission rate that was 25-35% higher than fixed allocation on average depending on the ratio of the number of channels over the number of users. The improvement is achieved at a cost of increased computation complexity through theoretical running time analysis, especially when the number of devices and channels increase.

Based on this experiment, several directions for further research can be pursued. First, the experiment used to compare the approaches was relatively small. Repeating it on a larger scale would offer more accurate and informative results. Additionally, it was shown that there is a trade-off between transmission rate and running time when using our approach over fixed allocation. To gain more insight into this trade-off and its implication on practicality, the allocation scheme should be tested under the conditions of a real wireless communications system.

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Introduction

Wireless communications utilize allocated frequency bands or “channels” on the radio frequency spectrum in order to transmit and receive signals. The radio frequency spectrum is a segment of the electromagnetic frequency spectrum containing frequencies between 30Hz and 300GHz.

The use of wireless devices is exploding in terms of the number of devices in use, how heavily they are used, and the amount of data needed to be transferred. Thus, the radio frequency spectrum is becoming more and more crowded. Since only one device can occupy a channel at a specific time and location, spectrum crowding poses a growing issue [2]. There are several approaches to alleviating spectrum crowding.

One approach is to expand the most heavily used bands of the radio frequency spectrum. The radio frequency spectrum is segmented into various frequency bands such as Very High Frequency (VHF), Ultra High Frequency (UHF), Super High Frequency (SHF). Each radio frequency band contains channels that are designated for specific types of wireless communications, like maritime radio (Low Frequency), TV (VHF, UHF), Wi-Fi (UHF, SHF), and more [1]. In expanding a frequency band, the spectrum is re-allocated so that the range of channels reserved for a particular service is widened. Thus, reallocating the spectrum according to the needs of wireless services can improve the efficient use of the spectrum. An example of spectrum re-allocation was the U.S. Federal Communications Commission’s spectrum auction in 2008, where carriers for cellular services bid on over 700 licenses that made up the 700 MHz frequency band [5]. This freed up some of the TV spectrum for other wireless service providers.

However, since the frequency spectrum is inherently finite, spectrum re-allocation or expansion will not provide a lasting solution to crowding.

Another approach to alleviating spectrum crowding is to increase the transmission efficiency so that more data can be transmitted over the frequency spectrum. With greater transmission efficiency, a given transmission will require fewer channels, so the frequency spectrum can accommodate more devices. This is the approach taken in our project and will be explained in greater detail later in the paper. This project aims to implement and test a spectrum allocation strategy that estimates the optimal user-channel assignment. Here, a “user” is any device capable of transmitting data wirelessly.

Background Information

Our approach aims to increase the data transmission efficiency by optimizing the maximum transmission rate of multiple users, each occupying multiple channels. This rate is calculated using a well-known formula from information theory on multiple channels for each user [6]. Equation 1 shows how the maximum transmission rate r_s is calculated for a user with signal power P over a set of channels s:

$$r_s = \sum_{j \epsilon s} \frac{1}{2} \log \left(1 + \frac{P_j}{N_j} \right) \quad (1)$$

where $\Sigma P_j = \mathrm{P}$ and N_j is the noise power for channel j.

The amount of signal power allocated to each channel is determined by the water filling algorithm. The water filling algorithm determines the distribution of signal power that optimizes transmission efficiency over several channels [7]. An example output of the algorithm is illustrated in Figure 1.

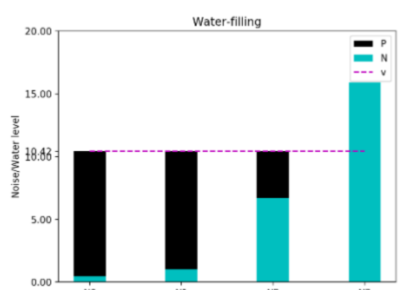

Fig. 1. Using the water filling method to split signal power P over channels with noise power $N_{0...3}$, keeping water-level v.

The spectrum allocation scheme most generally used is called fixed allocation. In this allocation scheme, the spectrum is split into equal size segments for each user. Users are then assigned to segments sequentially. Figure 2 shows an example of fixed allocation.

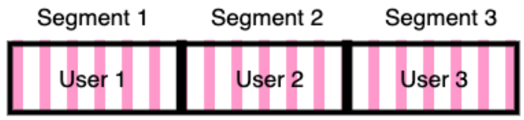

Fig. 2. Users are assigned to segments sequentially in fixed allocation.

Non-Sequential Approach

The amount of noise present in a channel varies by both channel and user [1]. Since a sequential allocation scheme does not account for varying noise levels, it is possible for a non-optimal assignment between users and channel segments to be chosen. Our approach improves upon transmission efficiency by assigning users to channel optimally.

We improve upon the fixed allocation scheme by calculating the maximum transmission rate if each user were to occupy each channel segment. The transmission rates are then used to populate a matrix which represents a formulation of the assignment problem. This can be solved to yield an optimal assignment between channel segments and users.

The Assignment Problem

The assignment problem is a classical applied mathematics problem in which one must find the minimum-cost assignment between N agents and M tasks, where $M = N$ and each agent-task combination may have a different cost, c_{nm}. In the optimal assignment, each agent must be assigned to exactly 1 task, and each task must be completed by exactly 1 user [3]. Figure 3 shows an example of how the assignment problem could be visualized.

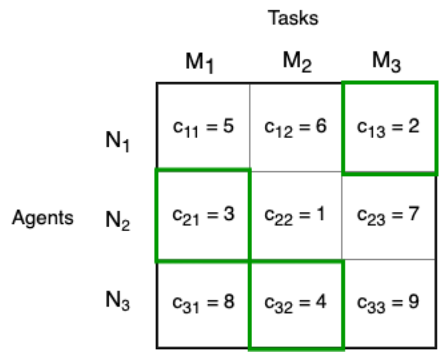

Fig 3. One example of the assignment problem and a solution. The problem is represented as a matrix, each row representing one agent and each column representing one task. The value in each cell represents the cost of the particular agent completing the particular task, and a minimum-cost solution to the problem is shown outlined in the green boxes.

Formulating the spectrum allocation problem as an assignment problem allows an optimal assignment to be found using the Hungarian Algorithm. This assignment represents a way to allocate users to channel segments on the frequency spectrum. The problem is formulated by treating channel segments S_i as agents, users P_i as tasks, and the negative of the maximum transmission rate r_{sp} as the reward of the assignment. $-r_{sp}$ is used because the assignment problem classically minimizes cost, instead of maximizing reward. Figure 4 visualizes this formulation of the assignment problem.

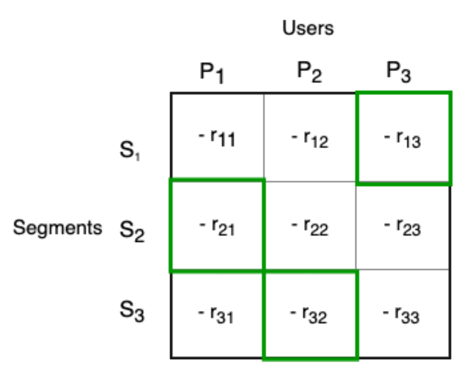

Fig 4. A formulation of spectrum allocation as an assignment problem in terms of channel segments, users, and maximum transmission rates for a given

user occupying a given channel segment. The green boxes represent a possible solution assignment.

The Hungarian Algorithm

The Hungarian Algorithm is an approach for finding an optimal solution to a classic *N x N* assignment problem. Several approaches to implementing the Hungarian Algorithm exist, differing in the way the assignment problem is represented. The problem can either be formulated as a matrix or a bipartite graph. Here, the problem is represented as a matrix. The algorithm generally works by performing admissible transformations on the matrix, marking particular cells, rows, and columns, and transforming the matrix according to these markings. Once each column is marked with an assignment, the marked coordinates will describe the minimum-cost assignment. Within the context of spectrum allocation, this corresponds to the segment-user assignment with the highest maximum transmission rate possible.

In our work, the Hungarian Algorithm has been implemented in Python as a seven-step algorithm, detailed in Jin Kue Wong's 1973 paper, "A new implementation of an algorithm for the optimal assignment problem: An improved version of Munkres' algorithm" [4].

Running Time Analysis

Pseudocode for the non-sequential allocation approach can be found below. From this, a theoretical worst-case runtime can be derived. The running time of the algorithm depends on both the number of channels and the number of users present. Our implementation of the water-filling and Hungarian algorithms run in $O(m^2)$ and $O(n^3)$ time sequentially, where *n* represents the number of users and *m* represents the number of channels. Since the nested for-loop contains a call to the water-filling algorithm, its running time is in O(n^2m^2). From this, it follows that the overall runtime is bounded by O(n^3+n^2m^2).

```
function allocation_assignment(channels, users):
    cost_matrix = create_matrix(users.length, users.length)
    segments = channels.split(users.length)
    for user in users:
        for segment in segments:
            max_rate = waterfill(user.power, segment)
            cost_matrix[user.index][segment.index] = max_rate
    assignment = hungarian_algorithm(cost_matrix)
    return assignment
```

Results

Our method of non-sequential allocation was evaluated against fixed allocation in terms of the overall maximum transmission rate. The overall rate is the sum of the maximum transmission rate for each user-segment pair in the chosen assignment. It is important to note that the maximum transmission rate yielded by non-sequential allocation will always be at least as high as the rate from fixed allocation. This is because the Hungarian algorithm considers all possible segment-user assignments, while fixed allocation only considers one.

To compare the approaches in a systematic way, we averaged the transmission rate yielded by each approach over 100 different inputs. The allocation methods were evaluated at several problem sizes: 16, 64 and 256 channels over 4 users. At each problem size, the approaches were compared by using a set of 100 distinct inputs, though the input set varied between problem sizes. The input consists of randomly generated signal power values for each user and randomly generated noise levels for each user and channel. Table I contains the experiment results.

Table I

	16 channels	**64 channels**	**256 channels**
Fixed Allocation (bits/transmission)	16.3724	36.3257	65.3738
Non-Sequential Allocation (bits/transmission)	20.3649	45.8702	88.8217
% Increase from Fixed to Non-Sequential	24.35%	26.27%	35.87%

Table I. The average maximum transmission rate of the assignment chosen using fixed allocation versus non-sequential allocation. The percent increase in average transmission rate between fixed and non-sequential allocation is also displayed.

The two allocation methods can also be compared in terms of worst-case runtime. Recall that the non-sequential approach runs in O(n^3+n^2m^2) time, where *n* is the number of users and *m* is the number of channels. Since fixed allocation simply splits the channels into *n* segments and assigns each user to a segment, it runs in O(*n*) time. This indicates that while the non-sequential method can yield a result that is up to 35% more efficient than fixed allocation, it will take much longer to compute an assignment as the input size increases. However, it is still much faster than an exhaustive search for the optimal assignment, which takes O(*n!*) time.

Discussion

We have presented an alternative method of frequency spectrum allocation which utilizes water-filling and the Hungarian Algorithm to determine an optimum user-channel segment assignment. The performance of our non-sequential method has also been evaluated in terms of maximum transmission rate and runtime.

Based on the results of the performance evaluation, several directions for further research can be considered. First, the systematic experiment used to compare the approaches was relatively small and could be repeated on a larger scale. More accurate results could be collected if a larger number of test inputs were used and the set of test inputs were the same for each problem size.

Additionally, it was shown that there is a trade-off between transmission rate and runtime when using non-sequential allocation over fixed allocation. To gain more insight into this trade-off and its implication on the feasibility of our approach, it will be important to test the allocation scheme under conditions of a real wireless communications system.

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